

# Breathing mode energy and nuclear matter incompressibility coefficient within relativistic and non-relativistic models

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Received: 1 October 2004 / Revised version: 4 November 2004 /

Published online: 11 April 2005 – © Società Italiana di Fisica / Springer-Verlag 2005

**Abstract.** Seemingly large differences ( $\sim 20\%$ ) in the value of the nuclear matter incompressibility coefficient  $K$  obtained from relativistic and non-relativistic models have been systematically investigated. For an appropriate comparison with the relativistic mean-field (RMF) based random phase approximation (RPA) calculations, we obtain the parameters of the Skyrme force used in the non-relativistic model by adopting the same experimental data and procedure employed in the determination of the NL3 parameter set of the effective Lagrangian used in the RMF model. Our investigation suggests that the discrepancy between the values of  $K$  predicted by the relativistic and non-relativistic models is less than 10%.

**PACS.** 21.65.+f Nuclear matter – 24.30.Cz Giant resonances – 21.60.Jz Hartree-Fock and random-phase approximations

## 1 Introduction

The Hartree-Fock (HF) based random phase approximation (RPA) provides a microscopic description of the nuclear compression modes such as the isoscalar giant monopole resonance (ISGMR), also referred to as the breathing mode. The centroid energy  $E_0$  of the ISGMR allows us to determine the value of nuclear matter incompressibility coefficient  $K$  which plays an important role in understanding a wide variety of phenomena. Currently, the uncertainty in the experimental data [1] for the  $E_0$  in heavy nuclei is  $\sim 0.1$ – $0.3$  MeV. The uncertainty  $\delta E_0$  associated with  $E_0$  is approximately related to the uncertainty  $\delta K$  in  $K$  by  $\frac{\delta K}{K} = 2\frac{\delta E_0}{E_0}$ , the value of  $\delta K$  is only about 10 MeV, for  $K = 250$  MeV and  $E_0 = 14.17 \pm 0.28$  MeV for the  $^{208}\text{Pb}$  nucleus. The theoretical scenario for  $K$  is [2,3],

$$K = \begin{cases} 250\text{--}270 \text{ MeV} & \text{Relativistic models,} \\ 205\text{--}212 \text{ MeV} & \text{Non-relativistic models.} \end{cases} \quad (1)$$

In eq. (1), the “relativistic” model refers to the calculations carried out using an effective Lagrangian which describes the nucleon-nucleon interaction through exchange of  $\sigma$ ,  $\omega$  and  $\rho$  mesons. Whereas, the “non-relativistic” models refer to the calculations carried out using a Skyrme type nucleon-nucleon effective interaction. The difference  $K[\text{Relativistic}] - K[\text{Non-relativistic}] \approx 50$ – $60$  MeV was

claimed to be due to the model dependence [3]. Some preliminary work to resolve this discrepancy was presented in ref. [4]. Our motivation is to investigate systematically the issue of the model dependence of  $K$  [5].

## 2 New Skyrme parameters (SK255)

As a first step toward resolving the issue of the model dependence of  $K$  we would like to match the typical differences between the relativistic and non-relativistic mean-field models. Most of the mean-field calculations carried out using the Skyrme interaction (non-relativistic) and the effective Lagrangian (relativistic) differ in the ways the contributions from the center-of-mass motion, Coulomb interaction and density dependence of the symmetry energy are dealt with [5].

For an appropriate comparison with the relativistic mean-field (RMF) based RPA calculations, we obtain the parameters of the Skyrme force used in the non-relativistic model by adopting the same experimental data and procedure employed in the determination of the NL3 parameter set of the effective Lagrangian used in the RMF model. In table 1 we have compared the nuclear matter properties obtained for the SK255 interaction with those for the NL3 parameter. Similar results for the binding energies and charge rms radii are presented in table 2 along with the experimental data.

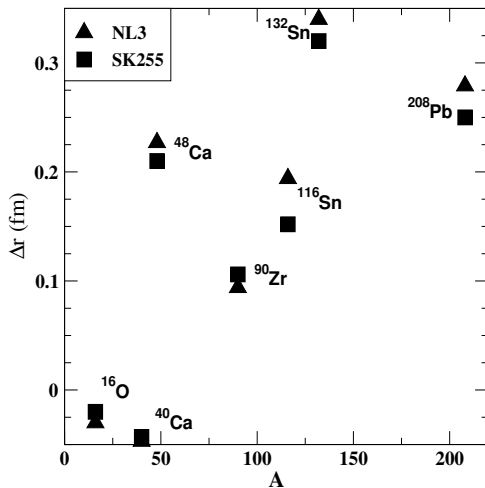
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**Table 1.** Comparison of the nuclear matter properties for the SK255 and NL3 interactions.

	NL3	SK255
$E/A$ (MeV)	-16.30	-16.33
$K_{nm}$ (MeV)	271.76	254.96
$\rho_0$ (fm $^{-3}$ )	0.148	0.157
$J$ (MeV)	37.4	37.4
$L$ (MeV)	118.5	95.0

**Table 2.** Binding energy and charge rms radii for the SK255 and NL3 interactions.

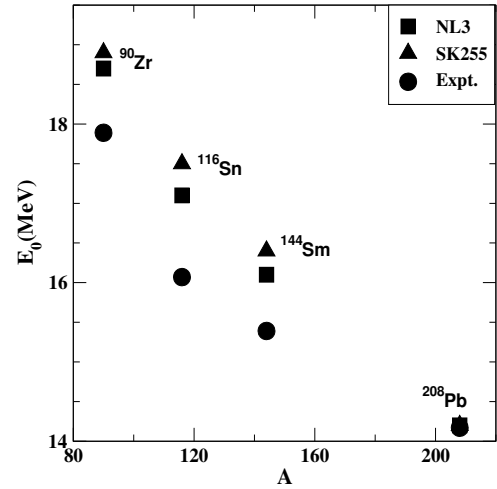
Nuclei	Binding energy (MeV)			rms charge radii (fm)		
	Expt.	NL3	SK255	Expt.	NL3	SK255
$^{16}\text{O}$	-127.6	-128.8	-128.1	2.73	2.73	2.81
$^{40}\text{Ca}$	-342.1	-342.0	-342.5	3.45	3.47	3.50
$^{48}\text{Ca}$	-416.0	-415.2	-413.9	3.45	3.47	3.53
$^{90}\text{Zr}$	-783.9	-782.6	-783.3	4.26	4.29	4.29
$^{116}\text{Sn}$	-988.7	-987.7	-984.5	4.63	4.61	4.62
$^{132}\text{Sn}$	-1102.9	-1105.4	-1100.0		4.71	4.73
$^{208}\text{Pb}$	-1636.5	-1639.5	-1637.5	5.50	5.52	5.50

**Fig. 1.** Neutron skin *versus* mass number  $A$  for the SK255 and NL3 interactions.

### 3 Neutron skin and breathing mode energy

In fig. 1 we display our results for  $\Delta r = r_n - r_p$ , the difference between the neutron and proton rms radii (the so-called neutron skin). The values of  $\Delta r$  for the SK255 and NL3 interactions are quite close to each other. For instance, in the case of  $^{208}\text{Pb}$  nucleus  $\Delta r = 0.25$  (0.28) fm for the SK255 (NL3) interactions. Most of the calculations using a Skyrme interaction having the value of  $K \sim 210$  MeV yield  $\Delta r \sim 0.16$  fm. The value of  $\Delta r$  is mainly governed by the values of  $J$  and  $L$ . For the SK255 interaction we have taken exactly the same value of  $J$  as obtained for the NL3 interaction (see table 1).

In fig. 2 we display our results for the breathing mode energy obtained using the SK255 interaction and compare them with those obtained using the NL3 interactions. We see that the maximum difference between the values of  $E_0$  obtained from the parameter sets SK255 and NL3 is

**Fig. 2.** Comparison of the results for the breathing mode energies obtained using the SK255 and NL3 interactions.

$\sim 0.3$  MeV, which is on the level of the uncertainty associated with the experimental data for  $E_0$ .

### 4 Conclusions

We have analyzed in detail the claim that the nuclear matter incompressibility coefficient  $K$  deduced from the breathing mode energy calculated within the relativistic and non-relativistic based RPA models differ by about 20%. For a meaningful comparison, we have generated a set of the Skyrme parameters SK255 ( $K = 255$  MeV) by using the same procedure and the experimental data for the bulk properties of nuclei considered in ref. [6] for determining the NL3 parameterization ( $K = 272$  MeV) of an effective Lagrangian used in the relativistic mean-field models. We have used the SK255 interaction to calculate the breathing mode energies for the several nuclei. We find that our results for the breathing mode energies are quite close to those obtained for the NL3 parameters of the relativistic model. This clearly demonstrates that for appropriately calibrated relativistic and non-relativistic models, the difference in the value of  $K$  is less than 10%.

This work was supported in part by the US Department of Energy under grant No. DOE-FG03-93ER40773 and the National Science Foundation under grant No. PHY-0355200.

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